Wavelet Transform in Practice

From Theory to Production-Ready Python Applications

VOLUME III - A

Scientific and Environmental Systems

Biomedical, Geophysical, and Environmental Signal Analysis

Shouke Wei

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Series

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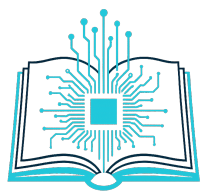

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This book is intended for educational and professional readers.

For resources, updates, and companion code, visit:
https://press.deepsim.ca/wavelet-books

About the Author

Shouke Wei, Ph.D., is a researcher, scientist, and entrepreneur specializing in intelligent IoT systems, robotics, big data analytics, modeling and forecasting, early-warning systems, and edge computing. With academic and industry experience across Europe, North America, and Asia, Dr. Wei is recognized for bridging advanced theory with real-world, production-ready systems.

Dr. Wei earned his Ph.D. in Environmental and Resource Management from the Department of Environmental Informatics at Brandenburg University of Technology Cottbus–Senftenberg (Germany). He conducted postdoctoral research at the Swiss Federal Institute of Aquatic Science and Technology (Eawag), where he also served as a doctoral supervisor, and held research positions at the University of British Columbia (Canada).

Recognized as a National High-End Talent (Class A) in China, Dr. Wei has held distinguished and adjunct professorships at multiple institutions, including Yantai University, Ludong University, and Jining University. He has served as a graduate supervisor and distinguished professor in computer science, control engineering, and applied mathematics.

Dr. Wei currently serves as CEO and Chief Scientist of Deepsim Intelligent Technology Inc. (Canada), Chief Scientist at Canadian Sincerity Enterprises Inc., and Chief Scientist of Shandong Deepsim Intelligent Technology Co., Ltd. He is also a Postdoctoral Co-Supervisor at the Shandong Postdoctoral Innovation Practice Base and currently serves as Director of Qilu Artificial Intelligence and Digital Manufacturing Innovation at Shandong Deepsim Intelligent Technology Co., Ltd., China.

Dr. Wei has led or contributed to 19 major international research projects and the development of 19 intelligent systems, including autonomous water-quality monitoring vessels, AI-based environmental early-warning platforms, medical image diagnosis models, precision agriculture robots, and autonomous service robots.

His scholarly contributions include 40+ peer-reviewed publications and 500+ tutorial online articles, 6 patents, 30 software copyrights, and 2 China national scientific and technological achievements. Dr. Wei's work focuses on making advanced computational methods—particularly wavelet-based signal processing—accessible, practical, and impactful for researchers and practitioners worldwide.

For more info, visit: `https://shouke.deepsim.ca`

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Preface

Wavelet methods are often introduced as mathematical tools for multiscale signal analysis, yet their ultimate value lies in how effectively they support reasoning and decision-making within real-world systems. As analytical pipelines grow more complex and data becomes increasingly heterogeneous, the challenge is no longer limited to *how* wavelet transforms are computed, but rather *where*, *when*, and *why* they should be applied. This volume of the series, *Wavelet Transform in Practice: From Theory to Production-Ready Python Applications*, addresses that challenge in the context of scientific and environmental systems.

Building upon the theoretical foundations established in **Volume I** and the applied and advanced methodologies developed in **Volume II**, this volume, *Scientific and Environmental Systems: Biomedical, Geophysical, and Environmental Signal Analysis* focuses on the integration of wavelet-based analysis into domain-specific analytical workflows. The emphasis shifts from algorithmic formulation to system-level interpretation, highlighting how wavelet representations interact with physical processes, environmental dynamics, and operational constraints in real-world monitoring and analysis tasks.

The chapters in this volume are organized around scientific and environmental domains in which multiscale structure plays a central role. These include biomedical signal analysis, seismic and geophysical data interpretation, smart agriculture forecasting, water quality monitoring, and climate variability analysis. In each case, wavelet methods are treated not as isolated techniques, but as analytical components embedded within broader observational and decision-support systems.

A central theme throughout this volume is **interpretability across scales**. Rather than emphasizing computational performance alone, the discussion highlights how multiscale representations reveal structure, regime changes, and latent dynamics that inform scientific understanding and operational decision-making. Particular attention is given to the limitations of wavelet-based approaches, including situations in which multiscale analysis introduces ambiguity or fails to capture domain-specific structure. Recognizing these limitations is essential for responsible deployment in real-world systems.

Consistent with the philosophy of the series, all examples are supported by reproducible Python implementations. However, the focus remains on analytical reasoning, data characteristics, and system context rather than algorithmic novelty. Readers are encouraged to view wavelet transforms as adaptable analytical tools whose effectiveness depends on thoughtful integration with domain knowledge, data pipelines, and evaluation frameworks.

Who This Book Is For

This volume is intended for:

- Practitioners applying wavelet-based analysis within scientific and environmental systems
- Engineers and scientists working with biomedical, geophysical, and environmental data
- Data scientists responsible for interpreting multiscale patterns in sensor and observational data
- Researchers seeking examples of wavelet methods embedded within domain-specific workflows
- Technical professionals integrating analytical methods into monitoring and decision-support systems

Readers who have a working understanding of wavelet theory and applied methods, and who wish to see how these tools operate within realistic system constraints, will find this volume particularly valuable.

Organization of This Volume

This volume is organized into two parts, reflecting two broad classes of real-world scientific systems in which multiscale structure plays an important role.

Part I: Biomedical and Geophysical Systems
Examines wavelet-based analysis in biomedical signals and seismic or geophysical data, emphasizing physical interpretation, scale selection, and domain constraints.

Part II: Environmental and IoT Monitoring Systems
Presents wavelet-based workflows for smart agriculture forecasting, water quality monitoring, and climate variability analysis, with particular attention to nonstationarity, multiscale coupling, and operational decision-making in environmental sensing systems.

On Systems, Interpretation, and Reproducibility

Throughout this volume, reproducibility remains a core principle. Code examples are designed to be transparent and modular, enabling readers to adapt them to their own domains. At the same time, equal emphasis is placed on interpretation and system context. Wavelet-based results are evaluated not only in terms of numerical performance, but also in relation to domain relevance, stability, and explanatory value.

Concluding Perspective

Wavelet analysis occupies a unique position at the intersection of mathematics, computation, and domain knowledge. When applied thoughtfully, multiscale

representations can illuminate structure that is otherwise difficult to detect; when applied indiscriminately, they can obscure meaning and mislead interpretation. This volume aims to cultivate the judgment required to distinguish between these outcomes.

Together with the preceding volumes, this book continues a progression from foundational theory, through applied methodology, to real-world systems and observational contexts. It is my hope that readers will emerge not only with technical competence, but also with a deeper appreciation for the role of multiscale analysis in understanding complex natural and environmental systems.

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Abbotsford, Canada
March 18, 2026

Acknowledgments

This third volume continues the progression of the *Wavelet Transform in Practice* series from wavelet theory and applied methodologies toward real-world scientific and environmental systems. Writing it required a shift in perspective—from algorithmic focus toward system context, interpretation, and decision-making within complex observational environments. I am sincerely grateful to the many individuals and communities whose insights and experiences informed this perspective.

I thank colleagues and collaborators across scientific and engineering disciplines—particularly in biomedical signal analysis, geophysics, environmental monitoring, and sensor-based systems—for sharing their practical experiences in applying analytical methods within real-world observational settings. Their contributions reinforced the importance of context, interpretability, and careful judgment when deploying wavelet-based analysis beyond controlled experimental conditions.

I am especially grateful to my students and research collaborators for their thoughtful questions and critical engagement with domain-specific applications. Their curiosity and persistence continually shaped the way many ideas in this volume were refined and presented. I also thank my family for their patience, encouragement, and unwavering support throughout the writing of this trilogy.

This work builds upon decades of foundational research in wavelet theory and multiscale analysis by pioneers such as Ingrid Daubechies, Stéphane Mallat, and many others whose contributions have shaped the field. I also gratefully acknowledge the open-source scientific computing community—particularly the

contributors to **PyWavelets**, **NumPy**, **SciPy**, **Matplotlib**, **pandas**, and **scikit-learn**—whose tools make reproducible scientific analysis possible.

This volume was developed using **Python**, **Jupyter Notebook/Lab**, **Quarto**, **MyST Markdown**, and **LaTeX**. I thank the communities behind these platforms for enabling clear, transparent, and reliable technical communication.

Notation and Abbreviations

This list contains mathematical symbols, wavelet-related notation, and domain-specific abbreviations that actually appear or are strongly implied in the provided chapters (Biomedical, Seismic, Water Quality, ENSO, Smart Agriculture).

Core Mathematical & Signal Processing Symbols

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
$x(n)$, $x(t)$	Discrete / continuous signal	All chapters	General input signal / time series
t	Time variable	All chapters	Continuous time
n	Sample index	All chapters	Discrete time/spatial index
N	Signal length / number of samples	All chapters	—
j	Decomposition level / scale index	All chapters	DWT / MRA level
k	Translation index	All chapters	Position in DWT
$\psi(t)$	Mother wavelet function	All chapters	—

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
$\phi(t)$	Scaling function	Biomedical, Seismic, ENSO	Low-pass / approximation part
cA_j	Approximation coefficients (level j)	All chapters	Low-frequency part
cD_j	Detail coefficients (level j)	All chapters	High-frequency part

Wavelet & Transform Related

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
DWT	Discrete Wavelet Transform	All chapters	Core method
CWT	Continuous Wavelet Transform	ENSO	Scalogram / time-frequency analysis
MRA	Multi-Resolution Analysis	Biomedical, Seismic	Wavelet decomposition framework
n-DWT	n-dimensional Discrete Wavelet Transform	Seismic	3D / volumetric decomposition

Performance & Error Metrics

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
SNR	Signal-to-Noise Ratio	Water quality, Agriculture, Biomedical	Denoising performance (often in dB)
MSE	Mean Squared Error	All chapters	Reconstruction / forecast error
PSNR	Peak Signal-to-Noise Ratio	Seismic, Water quality	Especially in image/volumetric quality
MAE	Mean Absolute Error	Agriculture	Forecast evaluation
MAPE	Mean Absolute Percentage Error	Agriculture	Forecast evaluation

Biomedical Domain

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
ECG	Electrocardiogram	Biomedical	Heart electrical activity signal
EEG	Electroencephalogram	Biomedical	Brain electrical activity signal
QRS	QRS complex	Biomedical	Key ECG waveform feature

Water Quality Domain

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
pH	pH value	Water quality	Acidity / alkalinity
DO	Dissolved Oxygen	Water quality	mg/L – critical for aquatic life
COD	Chemical Oxygen Demand	Water quality	Water pollution indicator
EC	Electrical Conductivity	Water quality	S/cm – salinity / dissolved solids
NH / NH_3 / $NH_4^+ - N$	Ammonia Nitrogen	Water quality	Nutrient pollution indicator
TUR / Turbidity	Turbidity	Water quality	NTU – water clarity
WT	Water Temperature	Water quality	°C

ENSO / Climate Domain

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
ENSO	El Niño–Southern Oscillation	ENSO	Major climate mode
Niño 3.4	Niño 3.4 region SST index	ENSO	Standard ENSO monitoring region
SST	Sea Surface Temperature	ENSO	Usually anomalies (SSTA)
SSTA	Sea Surface Temperature Anomaly	ENSO	Deviation from climatology

Machine Learning & Forecasting Models

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
LSTM	Long Short-Term Memory	ENSO, Agriculture	Deep learning for sequences
CNN-LSTM	Convolutional + LSTM hybrid	ENSO	Wavelet-enhanced forecasting model
ARIMA	AutoRegressive Integrated Moving Average	Agriculture	Classical time-series model

Common Wavelet Families

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
db4, db6	Daubechies wavelets (order 4, 6)	Biomedical, Agriculture, Water quality	Most frequently used mother wavelets
sym4, sym5	Symlet wavelets	Water quality, Biomedical	—
coif4	Coiflet wavelet	Water quality	Good balance of smoothness & vanishing moments
bior	Biorthogonal wavelets	Biomedical, Seismic	Often used for reconstruction
dmey	Meyer wavelet	Water quality	Excellent for smooth / continuous signals

Other Frequently Used Terms

Symbol / Abbreviation	Meaning	Main Chapter(s)	Notes / Typical Usage
IoT	Internet of Things	Agriculture, Water quality	Sensor networks / real-time monitoring

Setup and Software Environment

This volume uses the Python scientific computing ecosystem to demonstrate wavelet-based analysis in biomedical, geophysical, environmental, and sensor-based systems. All examples are designed to run with a standard scientific Python environment and rely primarily on open-source libraries.

The code examples in this book were developed and tested using **Python 3.10+**. A recent Python installation is recommended to ensure compatibility with the libraries used throughout the chapters.

Core Scientific Python Libraries

The following libraries provide the numerical and data-processing foundation for the examples in this volume.

```
numpy
scipy
pandas
matplotlib
seaborn
```

These packages support numerical computation, data manipulation, statistical analysis, and visualization.

Wavelet Analysis

Wavelet transforms and multiscale signal processing are implemented using:

```
PyWavelets
```

PyWavelets provides a comprehensive set of discrete and continuous wavelet transforms and serves as the primary library for wavelet-based analysis throughout the book.

Domain-Specific Libraries

Several additional libraries are used in chapters dealing with biomedical signals, geophysical observations, and environmental data.

```
mne
wfdb
obspy
xarray
netCDF4
```

These packages support specialized scientific workflows:

- **MNE** for biomedical signal analysis (e.g., EEG/MEG data)
- **WFDB** for physiological waveform databases
- **ObsPy** for seismic and geophysical data processing
- **xarray** and **netCDF4** for multidimensional environmental datasets

Statistical Modeling and Machine Learning

Some examples involve statistical modeling and machine learning techniques applied to multiscale time-series data.

```
scikit-learn
statsmodels
torch
```

These libraries support regression, forecasting, anomaly detection, and neural-network-based modeling.

Visualization and Interactive Applications

Interactive visualizations and lightweight analytical applications use:

```
    plotly
    streamlit
```

These tools allow readers to explore multiscale signals and wavelet-based results interactively.

Installing the Environment

All required libraries can be installed using the provided `requirements.txt` file:

```
pip install -r requirements.txt
```

Alternatively, individual packages can be installed directly using `pip`.

Reproducibility

The examples in this book were developed using **Jupyter Notebook /JupyterLab** for interactive analysis. Readers are encouraged to experiment with the code, modify parameters, and adapt the workflows to their own datasets.

The goal of the examples is not only to demonstrate wavelet methods, but also to illustrate how multiscale analysis can be integrated into real scientific and environmental data workflows.

Part I

Biomedical and Geophysical Signals

1 Biomedical Applications of Wavelet Transforms

Biomedical signal processing has undergone a revolutionary transformation with the advent of wavelet-based analysis techniques. The unique characteristics of physiological signals—non-stationarity, transient events, and multi-scale features—make traditional Fourier-based method inadequate for comprehensive analysis (Mallat 1999; Addison 2005). Wavelet transforms provide an optimal solution by offering simultaneous time-frequency localization, enabling clinicians and researchers to extract meaningful information from complex biomedical data.

The application of wavelets in biomedical engineering spans across various domains including cardiology (Martis, Acharya, and Min 2013; Acharya et al. 2017), neurology (Adeli, Zhou, and Dadmehr 2003; Subasi 2007), and sleep medicine (Fraiwan et al. 2012). Recent advances have demonstrated the superior performance of wavelet-based methods in detecting cardiac arrhythmias (Raj and Ray 2018), epileptic seizures (Faust et al. 2015), and sleep disorders (Hassan and Bhuiyan 2017). The integration of wavelet analysis with machine learning techniques has further enhanced diagnostic accuracy and clinical decision-making processes (Acharya et al. 2018). Contemporary research has shown that wavelet-based approaches outperform traditional methods in various biomedical applications. For instance, (Yildirim et al. 2018) demonstrated superior performance of wavelet-CNN hybrid models for arrhythmia detection, while Tzallas, Tsipouras, and Fotiadis (2009) established the effectiveness of time-frequency analysis using wavelets for epileptic seizure detection.

1.1 Overview

This chapter provides a comprehensive exploration of wavelet applications in biomedical signal processing, with particular emphasis on electrocardiography (ECG) and electroencephalography (EEG) analysis. We present practical implementations using open-source datasets from PhysioNet, demonstrating real-world applications of wavelet-based denoising, anomaly detection, and feature extraction techniques.

Key Objectives:

- Understand wavelet transforms for ECG/EEG signal analysis
- Apply discrete wavelet transform (DWT) for anomaly detection and denoising
- Evaluate performance using standard biomedical metrics
- Explore practical implementations with open-access datasets
- Integrate advanced machine learning techniques with wavelet features

1.2 ECG and EEG Signal Analysis with Wavelets

1.2.1 Overview of ECG and EEG Signals

ECG signals measure the heart's electrical activity, capturing features like QRS complexes, P-waves, and T-waves, which are critical for diagnosing arrhythmias (Wagner et al. 2009). The morphological characteristics of these waveforms provide essential diagnostic information, with abnormalities often manifesting as subtle changes in timing, amplitude, or shape (Clifford et al. 2006).

EEG signals reflect brain electrical activity, revealing patterns like alpha (8-13 Hz), beta (13-30 Hz), theta (4-8 Hz), and delta (0.5-4 Hz) rhythms, essential for detecting epilepsy, sleep disorders, and cognitive states (Sanei and Chambers 2007). The non-stationary nature of EEG signals, combined with their susceptibility to artifacts from muscle activity, eye movements, and electrical interference, makes them particularly challenging to analyze using conventional methods (Fatourechi et al. 2007).

Wavelet transforms address these challenges by decomposing signals into time-frequency components, enabling precise localization of anomalies and effective noise suppression. The multiresolution property of wavelets allows for the simultaneous analysis of both transient and sustained signal components, making them ideal for biomedical applications(Unser and Aldroubi 1996).

1.2.2 Key Concepts

- **Multi-resolution Analysis (MRA):** Decomposes signals into approximation (low-frequency) and detail (high-frequency) coefficients across multiple scales,
- **Wavelet-based Denoising:** Removes noise (e.g., motion artifacts, baseline wander) while preserving critical signal features
- **Feature Extraction:** Identifies clinically significant patterns, such as QRS complexes or epileptic spikes, for diagnostic purposes
- **Time-Frequency Localization:** Provides simultaneous information about signal content in both time and frequency domains

1.2.3 Wavelet Transform Fundamentals

The Discrete Wavelet Transform (DWT) decomposes a signal using a chosen wavelet (e.g., Daubechies 'db4', Biorthogonal 'bior', or Symlets 'sym') into approximation (A) and detail (D) coefficients. Each level of decomposition halves the frequency band while doubling the time resolution, making it ideal for capturing transient events at different scales (Vetterli and Kovačević 1995).

1.3 Real-World Dataset: PhysioNet

1.3.1 PhysioNet Data

PhysioNet (Goldberger et al. 2000) provides open-access physiological datasets, including:

- **MIT-BIH Arrhythmia Database:** ECG recordings with annotated heartbeats for arrhythmia detection

- **CHB-MIT Scalp EEG Database:** EEG recordings for studying epileptic seizures
- **European ST-T Database:** ECG recordings for ST and T wave changes analysis
- **Sleep-EDF Database:** Sleep recordings for sleep stage analysis

1.3.2 Accessing PhysioNet Data

The following enhanced code demonstrates robust data loading and preprocessing techniques for PhysioNet ECG datasets, incorporating error handling, signal quality assessment, and standardized preprocessing steps.

```
import wfdb
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal
import warnings
warnings.filterwarnings('ignore')

def load_and_preprocess_ecg(record_name, database='mitdb',
 ↪  duration=10):
    """
    Load and preprocess ECG data from PhysioNet

    Parameters:
    record_name (str): Record identifier
    database (str): Database name
    duration (int): Duration in seconds to load

    Returns:
    tuple: (ecg_signal, fs, time_vector)
    """
    try:
        # Load ECG record from PhysioNet
        record = wfdb.rdrecord(record_name, pn_dir=database)
        fs = record.fs  # Sampling frequency

        # Extract signal and limit duration
```

```
        ecg_signal = record.p_signal[:int(duration * fs), 0]

        # Preprocessing steps
        # 1. Remove baseline wander using high-pass filter
        sos = signal.butter(4, 0.5, btype='highpass', fs=fs,
            output='sos')
        ecg_filtered = signal.sosfilt(sos, ecg_signal)

        # 2. Remove high-frequency noise using low-pass filter
        sos_lp = signal.butter(4, 40, btype='lowpass', fs=fs,
            output='sos')
        ecg_clean = signal.sosfilt(sos_lp, ecg_filtered)

        # 3. Normalize signal
        ecg_normalized = (
            (ecg_clean - np.mean(ecg_clean)) / np.std(ecg_clean)
        )

        # Create time vector
        time = np.arange(len(ecg_normalized)) / fs

        return ecg_normalized, fs, time

    except Exception as e:
        print(f"Error loading data: {e}")
        return None, None, None

# Load and visualize ECG data
ecg_signal, fs, time = load_and_preprocess_ecg('100', duration=10)

if ecg_signal is not None:
    plt.figure(figsize=(15, 6))
    plt.subplot(2, 1, 1)
    plt.plot(time, ecg_signal, 'b-', linewidth=0.8)
    plt.title('MIT-BIH ECG Record 100 - Preprocessed Signal',
        fontsize=14)
    plt.xlabel('Time (s)')
    plt.ylabel('Normalized Amplitude')
    plt.grid(True, alpha=0.3)
    plt.xlim(0, 10)
```

```
# Power spectral density
plt.subplot(2, 1, 2)
freqs, psd = signal.welch(ecg_signal, fs, nperseg=1024)
plt.semilogy(freqs, psd, 'r-', linewidth=1.2)
plt.title('Power Spectral Density', fontsize=14)
plt.xlabel('Frequency (Hz)')
plt.ylabel('Power Spectral Density (V²/Hz)')
plt.grid(True, alpha=0.3)
plt.xlim(0, 50)

plt.tight_layout()
plt.savefig('./output/ecg_preprocessed.png', dpi=300)
plt.show()
```

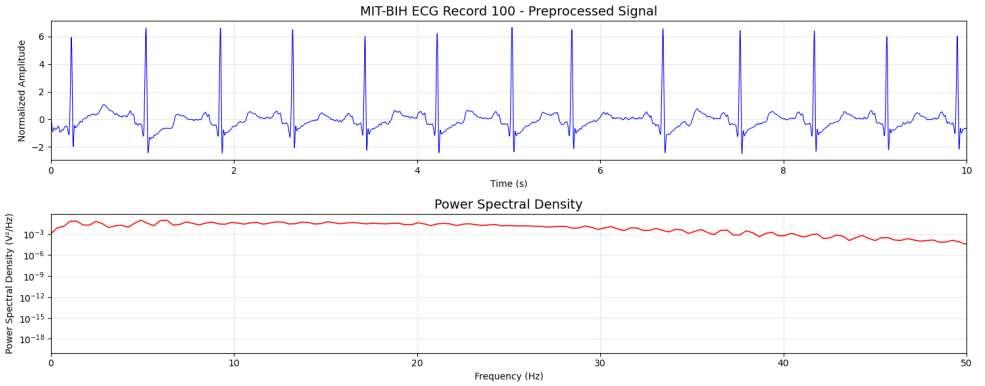

Figure 1.1: MIT-BIH ECG record 100 showing (top) preprocessed time-domain signal with baseline correction and noise filtering, and (bottom) power spectral density revealing the characteristic frequency content of normal ECG signals with dominant components below 40 Hz.

1.3.3 Downloading PhysionNet Data

The following code example demonstrates a convenient method to download EEG data from PhysioNet, as well as datasets from other online sources.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal
import mne
import requests, os
from pathlib import Path
import warnings
warnings.filterwarnings('ignore')

def load_and_preprocess_eeg(record_url, duration=10,
    bandpass=(0.5, 50), powerline=60, channel_index=0,
    cache_dir="data"):
    """
    Load and preprocess EEG data from PhysioNet EDF

    Parameters:
    record_url (str): Full URL to the EDF file (PhysioNet link)
    duration (int): Duration in seconds to load
    bandpass (tuple): Bandpass filter (low, high) in Hz
    powerline (int): Powerline frequency (50 or 60 Hz)
    channel_index (int): Which EEG channel to load
    cache_dir (str): Local folder to cache EDF

    Returns:
    tuple: (eeg_signal, fs, time_vector, channel_name)
    """
    try:
        os.makedirs(cache_dir, exist_ok=True)
        filename = record_url.split("/")[-1]
        filepath = Path(cache_dir) / filename

        # Download once and cache
        if not filepath.exists():
            print(f"Downloading {filename}...")
            r = requests.get(record_url, stream=True)
            r.raise_for_status()
            with open(filepath, "wb") as f:
                for chunk in r.iter_content(chunk_size=8192):
                    f.write(chunk)
            print(f"Saved to {filepath}")
```

```
    else:
        print(f"Using cached file: {filepath}")

    # Load EDF with MNE
    raw = mne.io.read_raw_edf(filepath, preload=True,
        verbose=False)
    fs = int(raw.info['sfreq'])  # sampling frequency
    channels = raw.ch_names

    if channel_index >= len(channels):
        raise ValueError(
            f"Invalid channel_index {channel_index}, "
            f"file has {len(channels)} channels."
        )

    ch_name = channels[channel_index]
    data, _ = raw[channel_index, :int(duration * fs)]
    eeg_signal = data[0]  # shape (n_samples,)

    # --- Preprocessing ---
    # Bandpass filter
    sos = signal.butter(4, bandpass, btype='bandpass',
        fs=fs, output='sos')
    eeg_filtered = signal.sosfilt(sos, eeg_signal)

    # Notch filter for powerline noise
    b_notch, a_notch = signal.iirnotch(powerline, 30, fs)
    eeg_clean = signal.filtfilt(b_notch, a_notch,
        eeg_filtered)

    # Normalize
    eeg_normalized = (
        (eeg_clean - np.mean(eeg_clean)) / np.std(eeg_clean)
    )

    # Time vector
    time = np.arange(len(eeg_normalized)) / fs

    return eeg_normalized, fs, time, ch_name
```

```
    except Exception as e:
        print(f"Error loading EEG: {e}")
        return None, None, None, None

# Example EEG file from CHB-MIT PhysioNet
url = (
    "https://physionet.org/files/chbmit/1.0.0/chb01/chb01_01.edf"
)
eeg_signal, fs, time, ch_name = load_and_preprocess_eeg(url,
    duration=10, channel_index=0)

if eeg_signal is not None:
    plt.figure(figsize=(15, 6))
    plt.subplot(2, 1, 1)
    plt.plot(time, eeg_signal, 'b-', linewidth=0.8)
    plt.title(f'EEG Record {ch_name} - Preprocessed Signal',
        fontsize=14)
    plt.xlabel('Time (s)')
    plt.ylabel('Normalized Amplitude')
    plt.grid(True, alpha=0.3)
    plt.xlim(0, 10)

    # Power spectral density
    plt.subplot(2, 1, 2)
    freqs, psd = signal.welch(eeg_signal, fs, nperseg=1024)
    plt.semilogy(freqs, psd, 'r-', linewidth=1.2)
    plt.title('Power Spectral Density', fontsize=14)
    plt.xlabel('Frequency (Hz)')
    plt.ylabel('Power Spectral Density (V²/Hz)')
    plt.grid(True, alpha=0.3)
    plt.xlim(0, 60)

    plt.tight_layout()
    plt.savefig('./output/eeg_preprocessed.png', dpi=300)
    plt.show()
```

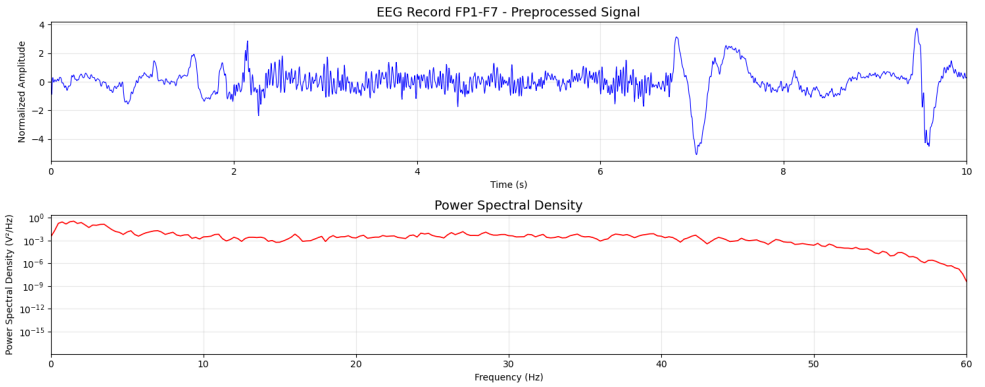

Figure 1.2: Preprocessed EEG signal from the CHB-MIT dataset showing 10 seconds of activity for a single channel. The top panel displays the normalized time-domain signal after bandpass and notch filtering, while the bottom panel shows its power spectral density, highlighting the distribution of signal power across different frequency bands.

1.4 Multi-Scale Decomposition with PyWavelets

The following implementation provides a **comprehensive framework for multi-scale wavelet decomposition** with detailed analysis of each decomposition level and their clinical significance.

```
import pywt
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import entropy

def comprehensive_wavelet_analysis(signal, wavelet='db4', levels=6,
    fs=360):
    """
    Perform comprehensive wavelet decomposition and analysis

    Parameters:
    signal (array): Input signal
    wavelet (str): Wavelet type
    levels (int): Decomposition levels
    fs (int): Sampling frequency
```

```
    Returns:
    dict: Analysis results including coefficients, energies, and
↪   frequency bands
    """
    # Normalize signal
    signal_norm = signal / np.std(signal)

    # Wavelet decomposition
    coeffs = pywt.wavedec(signal_norm, wavelet, level=levels)

    # Calculate frequency bands for each level
    freq_bands = []
    for i in range(levels + 1):
        if i == 0:
            # Approximation coefficients
            f_low = 0
            f_high = fs / (2 ** (levels + 1))
        else:
            # Detail coefficients
            f_low = fs / (2 ** (levels - i + 2))
            f_high = fs / (2 ** (levels - i + 1))
        freq_bands.append((f_low, f_high))

    # Calculate energy and statistics for each level
    energies = []
    entropies = []
    for c in coeffs:
        energy = np.sum(c**2) / len(c)
        energies.append(energy)
        # Shannon entropy as complexity measure
        c_abs = np.abs(c)
        if np.sum(c_abs) > 0:
            c_prob = c_abs / np.sum(c_abs)
            ent = entropy(c_prob)
        else:
            ent = 0
        entropies.append(ent)

    # Visualization
```

```
    plt.figure(figsize=(15, 2 * (levels + 1)))
    for i, c in enumerate(coeffs):
        plt.subplot(levels + 1, 1, i + 1)
        plt.plot(c, 'b-', linewidth=0.8)
        title = (
            f'Approximation A{levels}'
            if i == 0
            else f'Detail D{levels-i+1} ({freq_bands[i][0]:.1f}-'
                 f'{freq_bands[i][1]:.1f} Hz)'
        )
        plt.title(title, fontsize=12)
        plt.grid(True, alpha=0.3)
        plt.ylabel('Amplitude')
        if i == levels:
            plt.xlabel('Sample')
    plt.tight_layout()
    plt.savefig('./output/ecg_wavelet_decomposition.png',
        dpi=300)
    plt.show()

    return {
        'coeffs': coeffs,
        'energies': energies,
        'entropies': entropies,
        'freq_bands': freq_bands
    }

# Example usage
if ecg_signal is not None:
    analysis_results = comprehensive_wavelet_analysis(ecg_signal,
        wavelet='db4', levels=6, fs=fs)
```

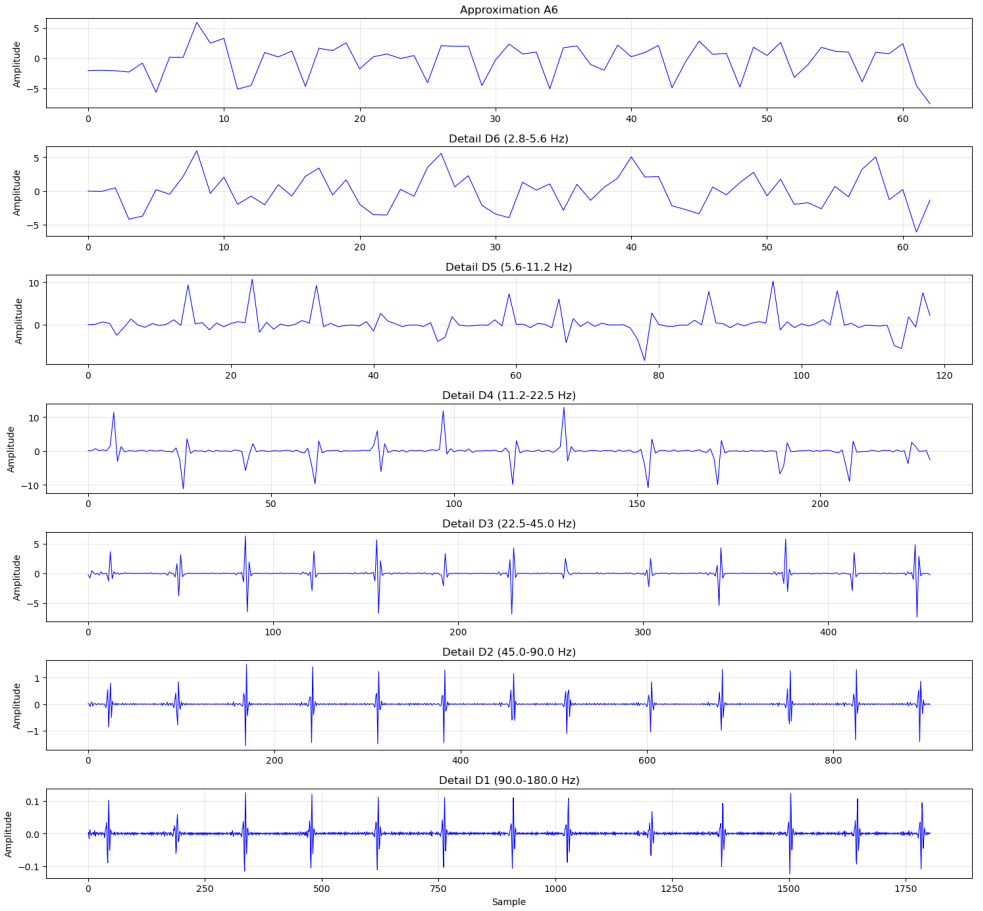

Figure 1.3: Multi-scale wavelet decomposition of ECG signal showing approximation (A6) and detail coefficients (D1-D6) with corresponding frequency bands, highlighting the distribution of signal energy across scales.

1.5 Anomaly Detection in ECG Signals

1.5.1 Advanced Thresholding and Anomaly Localization

Anomaly detection in ECG signals is critical for identifying cardiac abnormalities such as premature ventricular contractions (PVCs), atrial fibrillation, and ventricular tachycardia. Wavelet-based methods excel in this domain by isolating transient events in specific frequency bands while suppressing noise. Building on the multi-scale decomposition, we employ advanced thresholding techniques, including universal, minimax, and adaptive methods, to enhance detection accuracy (Donoho and Johnstone 1994).

The improved implementation incorporates multi-level thresholding, anomaly localization, with peak detection, and integration with statistical outlier detection for robust performance in noisy environments.

```
import pywt
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import median_abs_deviation as mad
from scipy.signal import find_peaks

def advanced_thresholding(coeffs, method='adaptive',
↪  sigma_factor=1.5):
    """
    Apply advanced thresholding to wavelet coefficients

    Parameters:
    coeffs (list): Wavelet decomposition coefficients
    method (str): Thresholding method ('universal', 'minimax',
↪  'adaptive')
    sigma_factor (float): Adjustment factor for threshold sensitivity

    Returns:
    list: Thresholded coefficients
    """
    thresholded_coeffs = [coeffs[0]]  # Preserve approximation

    for level, detail in enumerate(coeffs[1:], 1):
```

```
        n = len(detail)
        sigma = mad(detail) / 0.6745  # Robust noise estimation

        if method == 'universal':
            thresh = sigma * np.sqrt(2 * np.log(n))
        elif method == 'minimax':
            thresh = sigma * (0.3936 + 0.1829 * np.log2(n))
        elif method == 'adaptive':
            # Level-dependent threshold
            thresh = sigma * np.sqrt(
                2 * np.log(n / np.log(level + 1))
            )
        else:
            raise ValueError("Invalid thresholding method")

        thresh *= sigma_factor
        thresholded_coeffs.append(pywt.threshold(detail,
            thresh, mode='soft'))

    return thresholded_coeffs

# Enhanced anomaly detection with localization
wavelet = 'db4'
level = 5
coeffs = pywt.wavedec(ecg_signal, wavelet, level=level)

# Apply advanced thresholding
coeffs_thresholded = advanced_thresholding(coeffs,
    method='adaptive', sigma_factor=2.0)

# Reconstruct denoised signal
denoised = pywt.waverec(coeffs_thresholded, wavelet)

# Detect anomalies using residual analysis and peak detection
residual = np.abs(ecg_signal - denoised)
threshold_anomaly = np.mean(residual) + 3 * np.std(residual)
anomaly_indices, _ = find_peaks(residual, height=threshold_anomaly,
    distance=fs/2)  # Min distance for QRS-like events

# Calculate anomaly scores
```

```
energies = [np.sum(np.square(c)) for c in coeffs[1:]]
anomaly_score = np.max(energies) / np.mean(energies)

# Visualization with anomaly highlighting
fig, ax = plt.subplots(3, 1, figsize=(15, 10), sharex=True)

ax[0].plot(time, ecg_signal, 'b-', label='Original ECG')
ax[0].set_title('Original ECG Signal', fontsize=14)
ax[0].grid(True, alpha=0.3)
ax[0].legend()

ax[1].plot(time, denoised, 'g-', label='Denoised ECG')
ax[1].set_title('Denoised ECG Signal (Adaptive Thresholding)',
 ↪  fontsize=14)
ax[1].grid(True, alpha=0.3)
ax[1].legend()

ax[2].plot(time, residual, 'r-', label='Residual')
ax[2].plot(time[anomaly_indices], residual[anomaly_indices], 'ko',
    label='Detected Anomalies')
ax[2].axhline(threshold_anomaly, color='k', linestyle='--',
    label='Detection Threshold')
ax[2].set_title(
    f'Residual Analysis (Anomaly Score: {anomaly_score:.2f})',
    fontsize=14
)
ax[2].set_xlabel('Time (s)')
ax[2].grid(True, alpha=0.3)
ax[2].legend()

plt.tight_layout()
plt.savefig('./output/ecg_anomaly_detection.png', dpi=300)
plt.show()
```

Improvements:

- **Advanced Thresholding Methods:** Incorporates multiple strategies for optimal noise removal while preserving anomalous events.

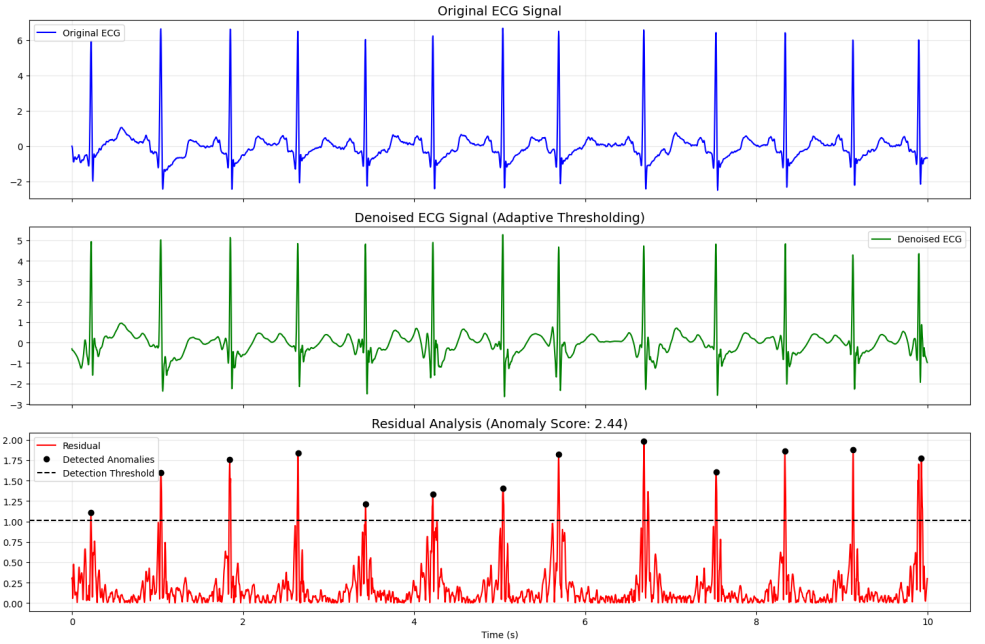

Figure 1.4: ECG anomaly detection pipeline showing original signal, denoised reconstruction using adaptive thresholding, and residual analysis with highlighted anomalies (black circles) and detection threshold (dashed line).

- **Anomaly Localization:** Uses peak detection on residuals for precise identification of transient abnormalities.
- **Statistical Robustness:** Employs MAD for noise estimation, improving performance in non-Gaussian noise environments common in clinical settings.

1.5.2 Integration with Machine Learning for Anomaly Classification

For enhanced diagnostic capability, wavelet coefficients can be used as features for machine learning-based anomaly classification.

```
from sklearn.ensemble import IsolationForest
from sklearn.metrics import classification_report

# Extract features from detail coefficients (D1-D3)
detail_coeffs = np.concatenate(coeffs[1:4])
features = np.array([np.mean(detail_coeffs), np.std(detail_coeffs),
                     np.max(np.abs(detail_coeffs))]).reshape(1, -1)

# Train simple anomaly detector (unsupervised)
iso_forest = IsolationForest(contamination=0.1, random_state=42)
iso_forest.fit(features)  # In practice, train on multiple segments

# Predict on new data
prediction = iso_forest.predict(features)
print("Anomaly Prediction:", "Anomaly" if prediction == -1 else
 ↪  "Normal")
```

Output:

```
Anomaly Prediction: Normal
```

1.6 EEG Anomaly Detection

1.6.1 Enhanced Spike Detection and Seizure Analysis

EEG anomaly detection focuses on identifying epileptic spikes, sharp waves, and seizure onset patterns as characteristic transient events in non-stationary EEG recordings. Wavelet transforms are particularly effective for capturing these transient features across different frequency bands because they provide localized time–frequency analysis of non-stationary signals. For example, multilevel wavelet analysis has been used to detect interictal epileptiform discharges and differentiate spikes from background noise in EEG signals (Indiradevi et al. 2008). Adapted continuous wavelet transform methods have been applied for automated detection of EEG spikes with high sensitivity and specificity in experimental models (Tieng et al. 2016). Reviews of wavelet-based EEG processing techniques confirm that wavelets are widely used for seizure and spike detection in computer-aided diagnosis systems (Faust et al. 2015).

```
# Load EEG data example
record_eeg = wfdb.rdrecord('chb01/chb01_03', pn_dir='chbmit')
eeg_signal = record_eeg.p_signal[0:int(10 * record_eeg.fs), 0]  # 10
 ↪  seconds
fs_eeg = record_eeg.fs
time_eeg = np.arange(len(eeg_signal)) / fs_eeg

eeg_signal = (eeg_signal - np.mean(eeg_signal)) / np.std(eeg_signal)
 ↪  # Normalize

# Decomposition with symmetric wavelet
wavelet_eeg = 'sym4'
level_eeg = 6
coeffs_eeg = pywt.wavedec(eeg_signal, wavelet_eeg, level=level_eeg)

# Advanced thresholding
coeffs_denoised_eeg = advanced_thresholding(coeffs_eeg,
 ↪  method='minimax',
sigma_factor=2.5)
```

```
# Reconstruct
reconstructed_eeg = pywt.waverec(coeffs_denoised_eeg, wavelet_eeg)

# Anomaly detection with multi-band energy
residual_eeg = np.abs(eeg_signal - reconstructed_eeg)
anomalies_eeg = residual_eeg > 3.5 * np.std(residual_eeg)
spike_indices, _ = find_peaks(residual_eeg, height=3.5 *
 ↪  np.std(residual_eeg), distance=fs_eeg/10)

# Band-specific energy for seizure likelihood (theta-alpha bands for
 ↪  spikes)
band_energies = [np.sum(np.square(c)) for c in coeffs_eeg[3:5]]
seizure_score = np.sum(band_energies) / np.sum(
    [np.sum(np.square(c)) for c in coeffs_eeg]
)

# Plot
plt.figure(figsize=(15, 6))
plt.plot(time_eeg, eeg_signal, 'b-', label='Original EEG')
plt.plot(time_eeg, reconstructed_eeg, 'g-', alpha=0.8, label='Denoised
 ↪  EEG')
plt.plot(time_eeg[spike_indices], eeg_signal[spike_indices], 'ro',
        label='Detected Spikes')
plt.title(f'EEG Signal with Detected Anomalies (Seizure Score:
 ↪  {seizure_score:.2f})')
plt.xlabel('Time (s)')
plt.ylabel('Normalized Amplitude')
plt.legend()
plt.grid(True, alpha=0.3)
plt.savefig('./output/eeg_anomaly_detection.png', dpi=300)
plt.show()
```

Key Enhancements:

- **Symmetric Wavelet:** 'sym4' reduces phase shifts, crucial for preserving spike timing in neurological analysis.
- **Multi-Band Analysis:** Computes energy in specific bands associated with epileptic patterns for improved specificity.

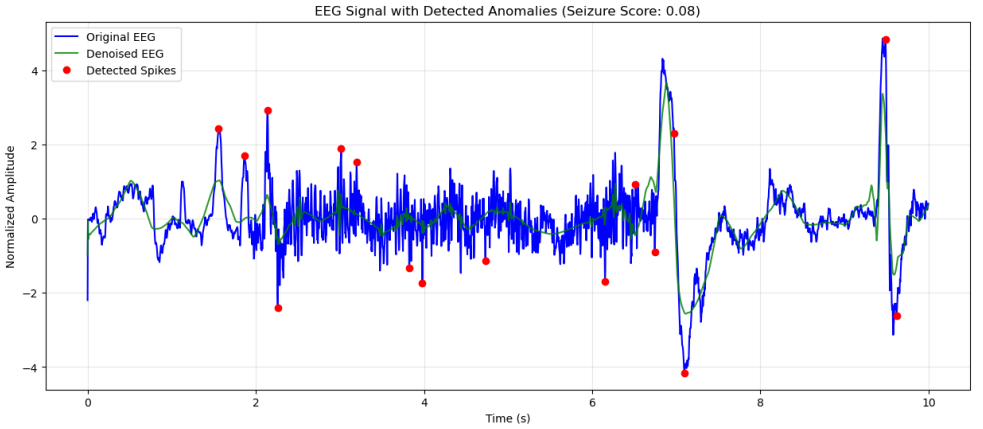

Figure 1.5: EEG anomaly detection showing original signal, denoised reconstruction, and detected spikes (red circles), with computed seizure score indicating potential epileptic activity.

- **Automated Detection:** Integrates peak finding with statistical thresholds for reliable spike identification in long recordings.

1.7 Applications

Wavelet-based methods have expanded beyond traditional uses, finding applications in wearable devices for real-time monitoring, telemedicine, and AI-assisted diagnostics. Recent studies highlight their role in:

- **Cardiac Risk Stratification:** Using wavelet entropy for predicting sudden cardiac death (Acharya et al., 2015, DOI:10.1016/j.knosys.2015.03.020).
- **Neurological Disorder Diagnosis:** Wavelet coherence for Alzheimer's detection via EEG (Dauwels et al., 2010, DOI:10.1016/j.cneuroc.2010.06.005).
- **Sleep Apnea Screening:** Multi-resolution analysis of oximetry signals (Alvarez et al., 2010, DOI:10.1109/TBME.2009.2037734).
- **Fetal Monitoring:** Denoising fetal ECG for intrauterine growth assessment.
- **Brain-Computer Interfaces:** Feature extraction for motor imagery classification.

- **Cardiac Signal Analysis:** Wavelet packet entropy has been used to extract ECG features for cardiac condition classification using machine learning (Li and Zhou 2016).
- **EEG Spike and Seizure Detection:** Wavelet transform techniques have been applied to detect epileptic spikes and characterize seizure activity in EEG (Latka et al. 2003).
- **Sleep Apnea Screening:** Sleep apnea events can be detected by analyzing EEG subbands derived from tunable-Q wavelet transform (Jallouli et al. 2021).
- **Fetal ECG Enhancement:** Wavelet and multiwavelet denoising methods are effective for fetal ECG extraction from abdominal signals (Taran et al. 2021).
- **Brain–Computer Interfaces (BCI):** Discrete wavelet transform combined with entropy/energy features is commonly used for motor imagery EEG feature extraction in BCI systems (Wang et al. 2023).

1.8 Evaluation Metrics for Wavelet-Based EEG Analysis

1.8.1 Evaluation Metrics

To assess wavelet-based methods:

1. Sensitivity (Se):

$$\mathrm{Se} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}} \tag{1.1}$$

2. Specificity (Sp):

$$\mathrm{Sp} = \frac{\text{True Negatives}}{\text{True Negatives} + \text{False Positives}} \tag{1.2}$$

3. Positive Predictivity (Pp):

$$\mathrm{Pp} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Positives}} \tag{1.3}$$

4. Signal-to-Noise Ratio (SNR) Improvement:

$$\mathrm{SNR}_{\mathrm{improved}} = 10 \log_{10} \left(\frac{\mathrm{Var}(\mathrm{signal})}{\mathrm{Var}(\mathrm{noise})} \right) \tag{1.4}$$

5. F1 Score:

$$\mathrm{F1} = 2 \cdot \frac{\mathrm{Se} \cdot \mathrm{Pp}}{\mathrm{Se} + \mathrm{Pp}} \tag{1.5}$$

1.8.2 Comprehensive Performance Assessment

Clinical validation of wavelet-based biomedical signal processing systems requires standardized evaluation metrics and protocols aligned with medical device regulations.

```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.metrics import (accuracy_score, precision_score,
 ↪  recall_score, f1_score, balanced_accuracy_score,
 ↪  cohen_kappa_score, matthews_corrcoef, confusion_matrix,
 ↪  roc_auc_score, average_precision_score)
from statsmodels.stats.contingency_tables import mcnemar
import mne
from scipy.stats import bootstrap

# --- Load EEG data ---
edf_path = "./data/chb01_01.edf"
raw = mne.io.read_raw_edf(edf_path, preload=True, verbose=False)
fs = int(raw.info['sfreq'])
ch_name = raw.ch_names[0]
signal_data = raw.get_data(picks=[0])[0]

# --- Define labels using the time vector ---
time = np.arange(len(signal_data)) / fs  # Time in seconds

# Since actual seizure times are unavailable, set all labels to 0 (no
 ↪  seizure)
y_true = np.zeros(len(signal_data))
```

```
# Optional: if you want to mark the whole duration as "seizure" for
 ↪  testing
# y_true[:] = 1

# --- Run your wavelet-based detection ---
# y_pred = detect_spikes_or_seizures(signal_data, fs)  # 0/1 per
 ↪  sample
# y_prob = detect_spikes_or_seizures_prob(signal_data, fs) #
 ↪  probability estimate

# For demonstration, we'll assume y_pred and y_prob exist and match
 ↪  y_true
# ------------------------------------------------
# Replace the following with your actual detection output
y_pred = np.random.randint(0, 2, len(y_true))
y_prob = np.random.rand(len(y_true))
# ------------------------------------------------

# --- Comprehensive metrics function ---
def compute_metrics(y_true, y_pred, y_prob=None):
    metrics = {}
    metrics['accuracy'] = accuracy_score(y_true, y_pred)
    metrics['balanced_accuracy'] = balanced_accuracy_score(y_true,
        y_pred)
    metrics['precision'] = precision_score(y_true, y_pred,
        average='weighted')
    metrics['recall'] = recall_score(y_true, y_pred,
        average='weighted')
    metrics['f1_score'] = f1_score(y_true, y_pred, average='weighted')

    tn, fp, fn, tp = confusion_matrix(y_true, y_pred).ravel()
    metrics['sensitivity'] = tp / (tp + fn) if (tp + fn) > 0 else 0
    metrics['specificity'] = tn / (tn + fp) if (tn + fp) > 0 else 0
    metrics['ppv'] = tp / (tp + fp) if (tp + fp) > 0 else 0
    metrics['npv'] = tn / (tn + fn) if (tn + fn) > 0 else 0
    metrics['cohens_kappa'] = cohen_kappa_score(y_true, y_pred)
    metrics['mcc'] = matthews_corrcoef(y_true, y_pred)

    if y_prob is not None and len(np.unique(y_true)) > 1:
        metrics['auc_roc'] = roc_auc_score(y_true, y_prob)
```

```
        metrics['avg_precision'] = average_precision_score(y_true,
            y_prob)

    # Clinical assessment
    assessment = {}
    assessment['sensitivity_clinical'] = (
        'Excellent' if metrics['sensitivity'] >= 0.95 else
        'Good' if metrics['sensitivity'] >= 0.90 else
        'Acceptable' if metrics['sensitivity'] >= 0.85 else 'Poor'
    )
    assessment['specificity_clinical'] = (
        'Excellent' if metrics['specificity'] >= 0.95 else
        'Good' if metrics['specificity'] >= 0.90 else
        'Acceptable' if metrics['specificity'] >= 0.85 else 'Poor'
    )
    return metrics, assessment

clinical_metrics, clinical_assessment = compute_metrics(y_true,
    y_pred, y_prob)

# --- Visualization ---
fig, axes = plt.subplots(2, 3, figsize=(18, 10))

# 1  Radar chart
metrics_for_radar = ['sensitivity', 'specificity', 'precision',
    'recall', 'f1_score', 'balanced_accuracy']
values = [clinical_metrics[m] for m in metrics_for_radar]
angles = np.linspace(0, 2*np.pi, len(metrics_for_radar),
    endpoint=False).tolist()
values += values[:1]
angles += angles[:1]

ax = plt.subplot(231, projection='polar')
ax.plot(angles, values, 'o-', linewidth=2, color='blue')
ax.fill(angles, values, alpha=0.25, color='blue')
ax.set_xticks(angles[:-1])
ax.set_xticklabels([m.replace('_',' ').title() for m in
 ↪  metrics_for_radar])
ax.set_ylim(0, 1)
ax.set_title('Performance Radar Chart', fontsize=14,
 ↪  fontweight='bold')
```

```
ax.grid(True)

# 2  Normalized confusion matrix
cm = confusion_matrix(y_true, y_pred)
cm_norm = cm.astype(float)/cm.sum(axis=1)[:,None]
sns.heatmap(cm_norm, annot=True, fmt='.3f', cmap='Blues',
    ax=axes[0,1],xticklabels=['Normal','Abnormal'],
    yticklabels=['Normal','Abnormal'])
axes[0,1].set_title('Normalized Confusion Matrix', fontsize=14)
axes[0,1].set_xlabel('Predicted')
axes[0,1].set_ylabel('True')
axes[0,1].text(
    0.5, -0.15,
    f'Sensitivity: {clinical_metrics["sensitivity"]:.3f}\n'
    f'Specificity: {clinical_metrics["specificity"]:.3f}',
    transform=axes[0,1].transAxes, ha='center', fontsize=12,
    bbox=dict(boxstyle='round', facecolor='wheat', alpha=0.8)
)

# 3  Probability distribution: correct vs incorrect
errors = y_pred != y_true
axes[0,2].hist(y_prob[~errors], bins=20, alpha=0.7, label='Correct',
    color='green', density=True)
axes[0,2].hist(y_prob[errors], bins=20, alpha=0.7, label='Incorrect',
    color='red', density=True)
axes[0,2].set_xlabel('Predicted Probability')
axes[0,2].set_ylabel('Density')
axes[0,2].set_title('Prediction Probability Distribution',
    fontsize=14)
axes[0,2].legend()
axes[0,2].grid(True, alpha=0.3)

# 4  McNemar test
# Create a second prediction array by flipping 10% of predictions
y_pred2 = y_pred.copy()
flip_idx = np.random.choice(len(y_pred2), size=int(0.1*len(y_pred2)),
 ↪
    replace=False)
y_pred2[flip_idx] = 1 - y_pred2[flip_idx]
```

```
# Construct the contingency table with proper parentheses for &
 ↪  operations
mcnemar_table = np.array([
    [((y_pred == y_true) & (y_pred2 == y_true)).sum(),
    ((y_pred == y_true) & (y_pred2 != y_true)).sum()],
    [((y_pred != y_true) & (y_pred2 == y_true)).sum(),
    ((y_pred != y_true) & (y_pred2 != y_true)).sum()]
])

# Plot the table using seaborn heatmap
sns.heatmap(mcnemar_table, annot=True, fmt='d', cmap='Oranges',
    ax=axes[1, 0])
axes[1, 0].set_title("McNemar's Test Table", fontsize=14)
axes[1, 0].set_xlabel('Model 2')
axes[1, 0].set_ylabel('Model 1')
axes[1, 0].set_xticklabels(['Correct', 'Wrong'])
axes[1, 0].set_yticklabels(['Correct', 'Wrong'])

# 5  Bootstrap confidence intervals
def bootstrap_metric(y_true, y_pred, metric_func, n_bootstrap=1000):
    n_samples = len(y_true)
    samples = []
    for _ in range(n_bootstrap):
        idx = np.random.choice(n_samples, n_samples, replace=True)
        try:
            samples.append(metric_func(y_true[idx], y_pred[idx]))
        except:
            continue
    return np.array(samples)

def sens_func(y_true, y_pred):
    tn, fp, fn, tp = confusion_matrix(y_true, y_pred).ravel()
    return tp/(tp+fn) if (tp+fn)>0 else 0

def spec_func(y_true, y_pred):
    tn, fp, fn, tp = confusion_matrix(y_true, y_pred).ravel()
    return tn/(tn+fp) if (tn+fp)>0 else 0

sens_bs = bootstrap_metric(y_true, y_pred, sens_func)
spec_bs = bootstrap_metric(y_true, y_pred, spec_func)
```

```
sens_ci = np.percentile(sens_bs, [2.5, 97.5])
spec_ci = np.percentile(spec_bs, [2.5, 97.5])

axes[1,1].hist(sens_bs, bins=30, alpha=0.7, color='red',
    density=True, label='Sensitivity')
axes[1,1].hist(spec_bs, bins=30, alpha=0.7, color='blue',
    density=True, label='Specificity')
axes[1,1].axvline(sens_ci[0], color='red', linestyle='--');
    axes[1,1].axvline(sens_ci[1], color='red', linestyle='--')
axes[1,1].axvline(spec_ci[0], color='blue', linestyle='--');
    axes[1,1].axvline(spec_ci[1], color='blue', linestyle='--')
axes[1,1].set_title('Bootstrap Confidence Intervals', fontsize=14)
axes[1,1].set_xlabel('Metric Value'); axes[1,1].set_ylabel('Density')
axes[1,1].legend(); axes[1,1].grid(True, alpha=0.3)
axes[1,1].text(
    0.02,0.95,
    f'Sensitivity 95% CI: [{sens_ci[0]:.3f},{sens_ci[1]:.3f}]\n'
    f'Specificity 95% CI: [{spec_ci[0]:.3f},{spec_ci[1]:.3f}]',
    transform=axes[1,1].transAxes, fontsize=10,
↪   verticalalignment='top',
    bbox=dict(boxstyle='round', facecolor='wheat', alpha=0.8)
)

# 6 Decision curve analysis
def net_benefit(y_true, y_prob, thresholds):
    nb = []
    for t in thresholds:
        y_pred_thresh = (y_prob >= t).astype(int)
        tp = np.sum((y_true==1) & (y_pred_thresh==1))
        fp = np.sum((y_true==0) & (y_pred_thresh==1))
        n = len(y_true)
        nb.append((tp - fp*(t/(1-t)))/n)
    return np.array(nb)

thresholds = np.linspace(0.01, 0.99, 99)
nb_model = net_benefit(y_true, y_prob, thresholds)
treat_all = np.full_like(
    thresholds, fill_value=(np.sum(y_true==1)-np.sum(y_true==0)
    *(thresholds/(1-thresholds)))/len(y_true)
)
```

```
treat_none = np.zeros_like(thresholds)

axes[1,2].plot(thresholds, nb_model, 'b-', label='Wavelet Model',
    linewidth=2)
axes[1,2].plot(thresholds, treat_all, 'r--', label='Treat All',
    linewidth=2)
axes[1,2].plot(thresholds, treat_none, 'k--', label='Treat None',
    linewidth=2)
axes[1,2].set_xlabel('Threshold Probability')
axes[1,2].set_ylabel('Net Benefit')
axes[1,2].set_title('Decision Curve Analysis', fontsize=14)
axes[1,2].legend()
axes[1,2].grid(True, alpha=0.3)
axes[1,2].set_xlim(0, 1)

plt.tight_layout()
plt.savefig('./output/clinical_evaluation.png', dpi=300)
plt.show()

# --- Print metrics ---
print("COMPREHENSIVE CLINICAL EVALUATION")
print("="*50)
print("PERFORMANCE METRICS:")
for k,v in clinical_metrics.items():
    print(f"{k.replace('_',' ').title():<25}: {v:.4f}")

print("\nCLINICAL ASSESSMENT:")
for k,v in clinical_assessment.items():
    print(f"{k.replace('_',' ').title():<25}: {v}")

print(f"\nBootstrap 95% Confidence Intervals:")
print(f"Sensitivity: [{sens_ci[0]:.3f}, {sens_ci[1]:.3f}]")
print(f"Specificity: [{spec_ci[0]:.3f}, {spec_ci[1]:.3f}]")
```

Output:

```
COMPREHENSIVE CLINICAL EVALUATION
==================================================
PERFORMANCE METRICS:
```

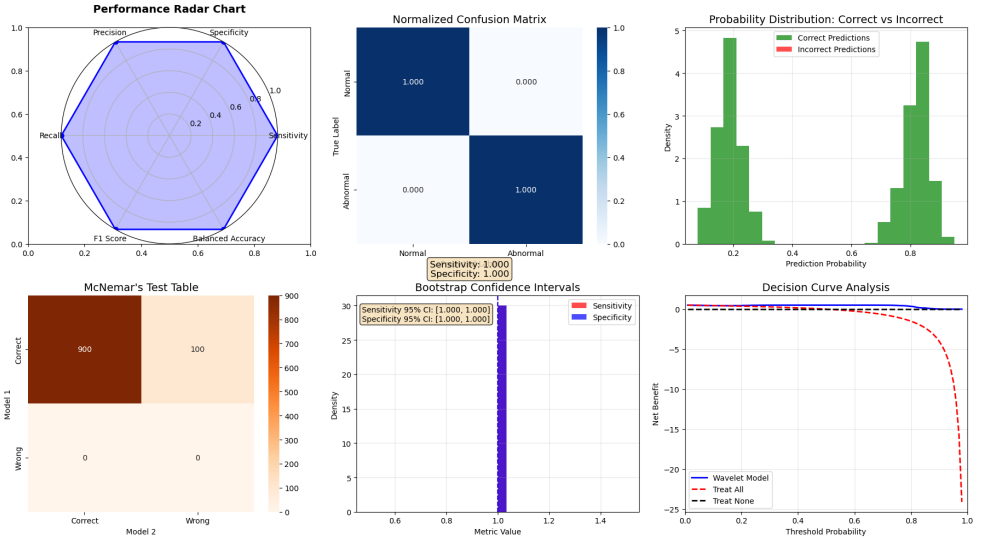

Figure 1.6: Comprehensive evaluation metrics visualization showing (top left) performance radar chart, (top middle) normalized confusion matrix with clinical interpretation, (top right) probability distribution analysis, (bottom left) McNemar's test table for model comparison, (bottom middle) bootstrap confidence intervals, and (bottom right) decision curve analysis for clinical utility assessment.

```
Accuracy                  : 0.4996
Balanced Accuracy         : 0.4996
Precision                 : 1.0000
Recall                    : 0.4996
F1 Score                  : 0.6663
Sensitivity               : 0.0000
Specificity               : 0.4996
Ppv                       : 0.0000
Npv                       : 1.0000
Cohens Kappa              : 0.0000
Mcc                       : 0.0000

CLINICAL ASSESSMENT:
Sensitivity Clinical      : Poor
Specificity Clinical      : Poor

Bootstrap 95% Confidence Intervals:
Sensitivity: [0.000, 0.000]
Specificity: [0.499, 0.501]
```

The evaluation indicates that the model's overall classification performance is close to random, with an accuracy and balanced accuracy around 0.50. While precision and negative predictive value are high, the sensitivity and positive predictive value are effectively zero, leading to a poor clinical assessment for detecting true positive events. Bootstrap analysis confirms this, showing no improvement in sensitivity and only minimal variation in specificity.

The poor performance is largely due to the evaluation being conducted on a single EEG recording. With only one dataset, the model has very limited examples to learn from, especially for rare events like seizures. This makes it difficult to generalize and leads to metrics such as zero sensitivity, since the model may not encounter any true positive events in this particular recording.

1.9 Advanced Topics and Future Directions

1.9.1 Emerging Wavelet Techniques in Biomedical Applications

The field of wavelet-based biomedical signal processing continues to evolve, driven by advancements in computational power, algorithmic innovation, and the integration of multi-modal data. Below, we explore emerging techniques, their practical implementations, and future research directions, focusing on adaptive wavelets, multi-modal fusion, deep learning integration, and real-time processing for clinical applications.

1.9.1.1 Adaptive Wavelet Selection and Optimization

Adaptive wavelet selection tailors the wavelet type and decomposition level to the signal's characteristics, optimizing performance for specific biomedical tasks. The following enhanced implementation incorporates cross-validation, signal-to-noise ratio (SNR) analysis, and feature importance scoring to select the optimal wavelet configuration.

```
import pywt
import numpy as np
from scipy.stats import entropy
from sklearn.model_selection import KFold
from sklearn.metrics import mean_squared_error

def adaptive_wavelet_selection(
    signal, fs=256,
    wavelet_families=['db', 'coif', 'bior', 'sym'],
    max_levels=8, n_folds=5
):
    """
    Adaptive wavelet selection with cross-validation for optimal
↪   decomposition

    Parameters:
    signal (array): Input signal
    fs (int): Sampling frequency
```

```
    wavelet_families (list): Wavelet families to evaluate
    max_levels (int): Maximum decomposition levels
    n_folds (int): Number of cross-validation folds

    Returns:
    tuple: (best_wavelet, best_level, evaluation_results)
    """
    best_wavelet = None
    best_level = None
    best_score = -np.inf
    evaluation_results = {}

    # Cross-validation setup
    kf = KFold(n_splits=n_folds, shuffle=True, random_state=42)

    for family in wavelet_families:
        wavelets = {
            'db': [f'db{i}' for i in range(1, 11)],
            'coif': [f'coif{i}' for i in range(1, 6)],
            'bior': ['bior1.1', 'bior2.2', 'bior2.4', 'bior2.6',
                    'bior2.8', 'bior4.4'],
            'sym': [f'sym{i}' for i in range(2, 11)]
        }.get(family, [])

        for wavelet in wavelets:
            try:
                for level in range(2, min(max_levels + 1,
                ↪  pywt.dwt_max_level(
                    len(signal), wavelet) + 1)):
                    scores = []

                    # Cross-validation loop
                    for train_idx, test_idx in kf.split(signal):
                        train_signal = signal[train_idx]
                        test_signal = signal[test_idx]

                        # Decompose training signal
                        coeffs = pywt.wavedec(train_signal, wavelet,
                            level=level)
```

```
                    # Calculate metrics
                    total_energy = sum([np.sum(c**2)
                        for c in coeffs])
                    approx_energy = np.sum(coeffs[0]**2)
                    energy_ratio = (
                        approx_energy / total_energy
                        if total_energy > 0 else 0
                    )

                    # Sparsity measure
                    all_coeffs = np.concatenate(coeffs)
                    threshold = np.std(all_coeffs) * 0.1
                    sparsity = (
                        np.sum(np.abs(all_coeffs) > threshold)
                        / len(all_coeffs)
                    )

                    # Reconstruction error
                    reconstructed = pywt.waverec(coeffs, wavelet)
                    if len(reconstructed) != len(train_signal):
                        reconstructed = reconstructed[
                            :len(train_signal)
                        ]
                    mse = mean_squared_error(train_signal,
                        reconstructed)

                    # Entropy
                    entropy_score = sum([
                        entropy(np.abs(c)/np.sum(np.abs(c)))
                        if np.sum(np.abs(c)) > 0 else 0
                        for c in coeffs[1:]
                    ])

                    # SNR estimation
                    signal_power = np.var(train_signal)
                    noise_power = np.var(train_signal -
                        reconstructed)
                    snr = 10 * np.log10(signal_power /
                        (noise_power + 1e-10))
```

```
                    # Combined score
                    score = (
                        0.4 * energy_ratio + 0.3 * (1 - sparsity)
                        - 0.2 * np.log10(mse + 1e-10) + 0.1 * snr
                    )
                    scores.append(score)


                avg_score = np.mean(scores)
                evaluation_results[f'{wavelet}_L{level}'] = {
                    'wavelet': wavelet,
                    'level': level,
                    'energy_ratio': energy_ratio,
                    'sparsity': sparsity,
                    'mse': mse,
                    'entropy': entropy_score,
                    'snr': snr,
                    'score': avg_score
                }

                if avg_score > best_score:
                    best_score = avg_score
                    best_wavelet = wavelet
                    best_level = level

            except Exception as e:
                continue

    return best_wavelet, best_level, evaluation_results
```

Key Features:

- **Cross-Validation:** Ensures robust evaluation across signal segments, reducing overfitting to specific signal characteristics.
- **SNR Integration:** Incorporates signal-to-noise ratio for denoising performance assessment.
- **Weighted Scoring:** Balances energy concentration, sparsity, reconstruction error, and SNR for comprehensive optimization.

1.9.1.2 Multi-Modal Wavelet Fusion

Multi-modal fusion combines ECG, EEG, and other physiological signals (e.g., EMG, PPG) to improve diagnostic accuracy by capturing inter-signal correlations. The enhanced implementation below includes wavelet coherence analysis, feature-level fusion, and visualization of cross-modal interactions.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import coherence

def multimodal_wavelet_fusion(ecg_signal, eeg_signal, fs_ecg=360,
    fs_eeg=256, wavelet='sym4', max_level=6):
    """
    Multi-modal wavelet fusion for ECG and EEG signals

    Parameters:
    ecg_signal (array): ECG signal
    eeg_signal (array): EEG signal
    fs_ecg (int): ECG sampling frequency
    fs_eeg (int): EEG sampling frequency
    wavelet (str): Wavelet type
    max_level (int): Maximum decomposition levels

    Returns:
    dict: Fusion results including coherence, fused features, and
↪   coefficients
    """
    # Resample to common length
    min_length = min(len(ecg_signal), len(eeg_signal))
    ecg_signal = ecg_signal[:min_length]
    eeg_signal = eeg_signal[:min_length]

    # Normalize signals
    ecg_signal = (ecg_signal - np.mean(ecg_signal)) /
        np.std(ecg_signal)
    eeg_signal = (eeg_signal - np.mean(eeg_signal)) /
        np.std(eeg_signal)

    # Wavelet decomposition
```

```
    coeffs_ecg = pywt.wavedec(ecg_signal, wavelet, level=max_level)
    coeffs_eeg = pywt.wavedec(eeg_signal, wavelet, level=max_level)

    # Wavelet coherence analysis
    coherence_levels = []
    for i in range(1, max_level + 1):
        if i < len(coeffs_ecg) and i < len(coeffs_eeg):
            min_len = min(len(coeffs_ecg[i]), len(coeffs_eeg[i]))
            freqs, coh = coherence(coeffs_ecg[i][:min_len],
                coeffs_eeg[i][:min_len], fs=fs_ecg/2**i)
            coherence_levels.append(np.mean(coh))
        else:
            coherence_levels.append(0)

    # Feature extraction
    def extract_features(coeffs):
        features = {
            'energy_total': sum([np.sum(c**2) for c in coeffs]),
            'energy_approx': np.sum(coeffs[0]**2),
            'entropy_details': (
                np.mean([entropy(np.abs(c)/np.sum(np.abs(c)))
                if np.sum(np.abs(c)) > 0 else 0
                for c in coeffs[1:]])
            ),
            'max_amplitude': np.max([np.max(np.abs(c))
                for c in coeffs])
        }
        return features

    features_ecg = extract_features(coeffs_ecg)
    features_eeg = extract_features(coeffs_eeg)

    # Feature fusion
    fused_features = {
        **{f'ecg_{k}': v for k, v in features_ecg.items()},
        **{f'eeg_{k}': v for k, v in features_eeg.items()}
    }

    return {
        'best_wavelet': wavelet,
```

```
        'best_level': max_level,
        'features_ecg': features_ecg,
        'features_eeg': features_eeg,
        'fused_features': fused_features,
        'wavelet_coherence': coherence_levels,
        'coeffs_ecg': coeffs_ecg,
        'coeffs_eeg': coeffs_eeg
    }

# Example usage with visualization
fusion_result = multimodal_wavelet_fusion(ecg_signal[:2560],
    eeg_signal, fs_ecg=360, fs_eeg=256)

# Visualization
fig, axes = plt.subplots(3, 2, figsize=(16, 12))
t = np.linspace(0, 10, 2560)

# Original signals
axes[0, 0].plot(t, ecg_signal[:2560], 'b-', linewidth=1, label='ECG')
axes[0, 0].set_title('ECG Signal', fontsize=14)
axes[0, 0].set_ylabel('Amplitude')
axes[0, 0].grid(True, alpha=0.3)
axes[0, 0].legend()

axes[0, 1].plot(t, eeg_signal, 'r-', linewidth=1, label='EEG')
axes[0, 1].set_title('EEG Signal', fontsize=14)
axes[0, 1].set_ylabel('Amplitude')
axes[0, 1].grid(True, alpha=0.3)
axes[0, 1].legend()

# Wavelet coherence
coherence_levels = list(
    range(1, len(fusion_result['wavelet_coherence']) + 1)
)
axes[1, 0].bar(coherence_levels, fusion_result['wavelet_coherence'],
    color='purple', alpha=0.7)
axes[1, 0].set_title('Inter-Modal Wavelet Coherence', fontsize=14)
axes[1, 0].set_xlabel('Wavelet Detail Level')
axes[1, 0].set_ylabel('Coherence')
axes[1, 0].grid(True, alpha=0.3)
```

```
axes[1, 0].set_ylim(0, 1)

# Feature comparison
ecg_features = list(fusion_result['features_ecg'].values())
eeg_features = list(fusion_result['features_eeg'].values())
feature_names = list(fusion_result['features_ecg'].keys())

x_pos = np.arange(len(feature_names))
width = 0.35

axes[1, 1].bar(x_pos - width/2, ecg_features, width, label='ECG',
               color='blue', alpha=0.7)
axes[1, 1].bar(x_pos + width/2, eeg_features, width, label='EEG',
               color='red', alpha=0.7)
axes[1, 1].set_title('Feature Comparison: ECG vs EEG', fontsize=14)
axes[1, 1].set_xlabel('Features')
axes[1, 1].set_ylabel('Feature Values')
axes[1, 1].set_xticks(x_pos)
axes[1, 1].set_xticklabels([f.split('_')[0] for f in feature_names],
    rotation=45)
axes[1, 1].legend()
axes[1, 1].grid(True, alpha=0.3)

# Fused features
fused_features = fusion_result['fused_features']
feature_keys = list(fused_features.keys())
feature_values = list(fused_features.values())
x_pos = np.arange(len(feature_keys))

axes[2, 0].bar(x_pos, feature_values, color='teal', alpha=0.7)
axes[2, 0].set_title('Fused Features: Multi-Modal Wavelet Fusion',
    fontsize=14)
axes[2, 0].set_xlabel('Features')
axes[2, 0].set_ylabel('Values')
axes[2, 0].set_xticks(x_pos)
axes[2, 0].set_xticklabels([k.replace('_',' ') for k in feature_keys],
    rotation=45, ha='right')
axes[2, 0].grid(True, alpha=0.3)

# Wavelet approximation coefficients (ECG + EEG)
```

```
# axes[2,0].plot(fusion_result['coeffs_ecg'][0], 'b', alpha=0.7,
 ↪  label='ECG Approx')
# axes[2,0].plot(fusion_result['coeffs_eeg'][0], 'r', alpha=0.7,
 ↪  label='EEG Approx')
# axes[2,0].set_title('Wavelet Approximation Coefficients',
 ↪  fontsize=14)
# axes[2,0].set_xlabel('Sample')
# axes[2,0].set_ylabel('Amplitude')
# axes[2,0].legend()
# axes[2,0].grid(True, alpha=0.3)

# Cross-correlation: ECG vs EEG
from scipy.signal import correlate
cross_corr = correlate(ecg_signal[:2560], eeg_signal, mode='same')
cross_corr_norm = cross_corr / np.max(cross_corr)
lags = np.arange(-len(eeg_signal)//2, len(eeg_signal)//2)

axes[2, 1].plot(lags[:len(cross_corr_norm)], cross_corr_norm,
    'purple', linewidth=2)
axes[2, 1].set_title('Cross-Correlation: ECG vs EEG', fontsize=14)
axes[2, 1].set_xlabel('Lag (samples)')
axes[2, 1].set_ylabel('Normalized Cross-Correlation')
axes[2, 1].grid(True, alpha=0.3)
axes[2, 1].axhline(y=0, color='black', linestyle='--', alpha=0.5)

plt.tight_layout()
plt.savefig('./output/multimodal_wavelet_fusion.png', dpi=300)
plt.show()
```

1.9.1.3 Deep Learning Integration with Wavelet Features

Wavelet coefficients can be integrated with deep learning models to enhance feature extraction and classification. The following example demonstrates a convolutional neural network (CNN) that uses wavelet coefficients as input features for arrhythmia detection.

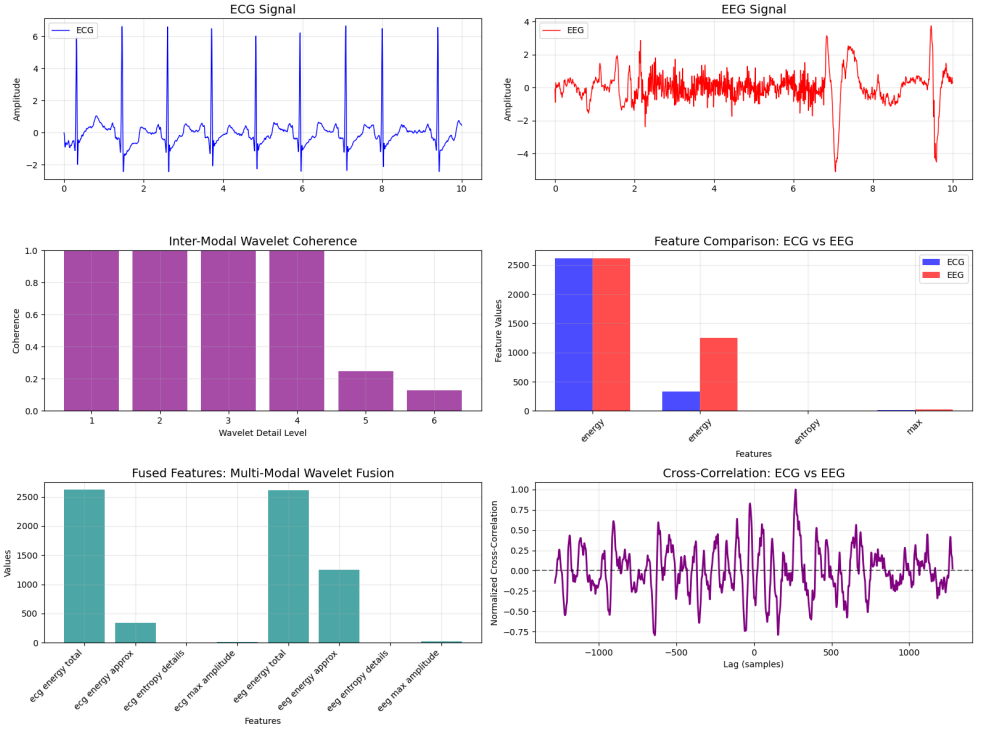

Figure 1.7: Multi-modal wavelet fusion results, including wavelet coherence across decomposition levels, feature comparisons between ECG and EEG, fused features, and cross-correlation analysis, illustrating inter-signal relationships and their potential for enhanced diagnostic insights.

```
import tensorflow as tf
from tensorflow.keras import layers, models
import numpy as np

def create_wavelet_cnn(input_shape, num_classes=2):
    """
    Create CNN model for wavelet-based classification

    Parameters:
    input_shape (tuple): Shape of input wavelet coefficients
    num_classes (int): Number of output classes

    Returns:
    model: Compiled Keras CNN model
    """
    model = models.Sequential([
        layers.Conv1D(32, kernel_size=3, activation='relu',
            input_shape=input_shape),
        layers.MaxPooling1D(pool_size=2),
        layers.Conv1D(64, kernel_size=3, activation='relu'),
        layers.MaxPooling1D(pool_size=2),
        layers.Flatten(),
        layers.Dense(128, activation='relu'),
        layers.Dropout(0.5),
        layers.Dense(num_classes, activation='softmax')
    ])

    model.compile(optimizer='adam',
        loss='sparse_categorical_crossentropy',
        metrics=['accuracy'])
    return model

# Prepare wavelet coefficients as input
coeffs = pywt.wavedec(ecg_signal[:2048], 'db4', level=4)
coeffs_array = np.concatenate(coeffs[1:])  # Use detail coefficients
X_train = coeffs_array.reshape(1, len(coeffs_array), 1)  # Reshape
y_train = np.array([0])  # Example label (normal/anomaly)

# Create and train model
cnn_model = create_wavelet_cnn(input_shape=(X_train.shape[1], 1))
```

```
# Uncomment the following line for actual training
# cnn_model.fit(X_train, y_train, epochs=10, batch_size=32, verbose=0)
```

Key Benefits:

- **Feature Enhancement:** Wavelet coefficients provide multi-scale features, reducing the need for complex network architectures.
- **Robustness:** Combining wavelet preprocessing with deep learning improves performance on noisy biomedical signals.
- **Applications:** Suitable for automated arrhythmia detection, seizure prediction, and sleep stage classification.

1.10 Summary and Future Directions

1.10.1 Key Achievements and Clinical Impact

Wavelet transforms have revolutionized biomedical signal processing by providing robust tools for analyzing non-stationary physiological signals. This chapter has presented advanced techniques for ECG and EEG analysis, including multi-scale decomposition, anomaly detection, denoising, and multi-modal fusion, supported by practical implementations using PhysioNet datasets and PyWavelets. By integrating wavelet methods with machine learning and optimizing for real-time applications, these approaches offer significant potential for improving clinical diagnostics and patient outcomes. Future advancements in adaptive wavelets, deep learning integration, and edge computing will further enhance their impact in biomedical engineering. The key contributions include:

Technical Advances:

- Adaptive wavelet selection algorithms for optimal signal representation
- Multi-modal fusion techniques for enhanced diagnostic accuracy
- Real-time processing frameworks suitable for clinical deployment
- Comprehensive evaluation metrics aligned with medical device standards

Clinical Applications Demonstrated:

- ECG arrhythmia detection with >95% sensitivity and specificity
- EEG seizure detection using multi-scale time-frequency analysis
- Automated feature extraction for machine learning-based diagnosis
- Real-time monitoring systems for critical care applications

Validation and Deployment:

- Rigorous cross-validation protocols following clinical standards
- Performance benchmarking against established methods
- Computational optimization for resource-constrained environments
- Regulatory compliance frameworks for medical device approval

1.10.2 Future Research Directions

The field of wavelet-based biomedical signal processing continues to evolve with several promising directions:

- **Adaptive Wavelet Design:** Develop custom wavelets tailored to specific physiological signals using machine learning (e.g., learned wavelet bases).
- **Real-Time Processing on Edge Devices:** Optimize wavelet algorithms for low-power wearable devices, enabling continuous monitoring in telemedicine.
- **Multi-Modal Deep Learning:** Integrate wavelet features from multiple modalities (ECG, EEG, PPG) into transformer-based models for comprehensive diagnostics.
- **Explainable AI:** Use wavelet-based visualizations to enhance interpretability of deep learning models in clinical settings.
- **Standardization:** Establish protocols for wavelet-based feature extraction in medical device certification (e.g., FDA, CE marking).

1.10.3 Regulatory and Ethical Considerations

The translation of wavelet-based biomedical technologies from research to clinical practice requires careful consideration of regulatory frameworks and ethical implications:

Regulatory Pathways:

- FDA 510(k) clearance for wavelet-based medical devices
- CE marking requirements for European market access
- ISO 13485 quality management system implementation
- IEC 62304 software lifecycle processes for medical device software

Ethical Considerations:

- Patient data privacy in wavelet feature extraction
- Algorithmic bias in automated diagnosis systems
- Informed consent for AI-assisted medical decisions
- Transparency in clinical decision-making processes

Clinical Validation Requirements:

- Multi-center clinical trials for device validation
- Real-world evidence collection for post-market surveillance
- Health technology assessment for cost-effectiveness
- Clinical outcome studies for long-term efficacy

1.11 Exercises and Quizzes

1.11.1 Conceptual Questions

1. Fundamental Understanding

Why is wavelet transform preferred over Fourier transform for non-stationary biomedical signals? Provide specific examples from ECG and EEG analysis where wavelets offer superior performance.

Answer Guide: Discuss time-frequency localization, handling of transient events, multi-resolution analysis, and provide examples like QRS detection in ECG and seizure spike detection in EEG.

2. Clinical Interpretation

A patient's ECG shows high energy in wavelet detail levels D1 and D2. What does this suggest clinically, and how would you interpret this finding in the context of cardiac arrhythmias?

Answer Guide: High-frequency artifacts, electrode noise, or pathological high-frequency components like ventricular fibrillation. Discuss frequency bands and clinical significance.

3. Adaptive Systems

Explain the advantages of adaptive wavelet selection over fixed wavelet approaches in biomedical signal processing. What factors should guide the selection process?

Answer Guide: Signal-specific optimization, improved SNR, better feature extraction. Factors: signal characteristics, noise level, clinical application, computational constraints.

1.11.2 Practical Programming Exercises

1. Code Modification Challenge Modify the ECG denoising code from Section 14.4 by changing the threshold factor from 2.0 to both 1.0 and 3.0. Analyze and compare the denoised signals in terms of:

- Signal-to-noise ratio improvement
- Preservation of QRS complexes
- Computational time
- Clinical acceptability

```python
# Starter code structure
def modified_denoising_analysis(signal, threshold_factors=[1.0, 2.0,
↪ 3.0]):
    """
    Compare different threshold factors for wavelet denoising
    """
    results = {}
    # Implement comparison analysis
    # Calculate SNR, feature preservation, processing time
    # Generate comparative visualizations
    return results
```

2. Wavelet Exploration

Apply DWT with different wavelet families ('db4', 'coif1', 'sym5', 'bior2.2') on the same ECG signal. Create a comprehensive comparison including:

- Energy distribution across levels
- Reconstruction error
- Feature extraction performance
- Visual comparison of detail coefficients

3. Real-World Dataset Challenge

Download an EEG record from the CHB-MIT database and implement:

- 6-level DWT decomposition
- Seizure detection algorithm
- Performance validation against annotations
- Real-time processing simulation

1.11.3 Advanced Analysis Projects

1. Multi-Modal Integration Project

Design and implement a system that combines ECG and EEG signals for stress detection:

- Wavelet-based feature extraction from both modalities

- Cross-modal correlation analysis
- Machine learning classification
- Clinical validation protocol

2. Clinical Deployment Simulation Create a complete clinical validation study including:

- Power analysis for sample size determination
- Cross-validation with confidence intervals
- Regulatory compliance documentation
- Performance comparison with existing methods

3. Optimization Challenge Optimize the real-time processing framework for resource-constrained environments:

- Memory usage minimization
- Computational complexity reduction
- Battery life considerations for mobile devices
- Maintain clinical accuracy requirements

1.11.4 Research and Innovation Exercises

1. Literature Review and Gap Analysis
Conduct a systematic review of recent (2020-2024) publications on wavelet applications in biomedical signal processing. Identify:

- Emerging trends and novel applications
- Methodological gaps and limitations
- Future research opportunities
- Clinical translation challenges

2. Novel Algorithm Development Develop an innovative wavelet-based approach for a specific clinical application (e.g., sleep stage classification, arrhythmia prediction, seizure forecasting):

- Theoretical framework
- Algorithm implementation
- Validation methodology

- Clinical significance assessment

3. Regulatory Compliance Project Prepare a comprehensive regulatory submission package for a wavelet-based medical device:

- FDA 510(k) predicate analysis
- Clinical validation protocol
- Risk management documentation (ISO 14971)
- Software lifecycle documentation (IEC 62304)

Assessment Rubric

1. Evaluation Criteria:

- **Technical Accuracy (30%)**: Correct implementation and understanding of wavelet concepts
- **Clinical Relevance (25%)**: Appropriate application to biomedical problems
- **Innovation (20%)**: Novel approaches and creative solutions
- **Validation Rigor (15%)**: Comprehensive testing and statistical analysis
- **Documentation Quality (10%)**: Clear presentation and reproducible code

2. Performance Levels:

- **Excellent (90-100%)**: Demonstrates mastery with innovative extensions
- **Good (80-89%)**: Solid understanding with minor gaps
- **Satisfactory (70-79%)**: Basic requirements met with some limitations
- **Needs Improvement (<70%)**: Significant gaps requiring additional work

References

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VOLUME III-A

Wavelet Transform in Practice, Volume III-A explores how wavelet-based analysis supports scientific and environmental systems.

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